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AN EXPLICIT SIMULATION OF OVERLAND FLOW

by

159

GLENDON TAYLOR STEVENS, JR., 1927-

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## AN EXPLICIT SIMULATION OF OVERLAND FLOW

By Glendon T. Stevens, Jr.<sup>1</sup> A.M. ASCE

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KEY WORDS: hydrograph; mathematical model; finite difference; explicit; convergence; hydraulics; overland flow; boundary conditions; computer

ABSTRACT: Computer simulation of overland flow requires a mathematical model which converges to a true solution. This paper presents a method, based on measurable physical parameters, which insures convergence of a fixed grid explicit numerical solution. Analysis of a wide range of overland flow plane parameters has resulted in an equation of the form  $\Delta t = 0.2 \Delta X / S_0$ , for the time step increment. This equation provides values for  $\Delta t$  that will insure stability and convergence of the fixed grid explicit numerical method of solving the spatially varied unsteady flow equations. The effect of subarea shape on the rising limb of the overland flow hydrograph is presented for three different configurations. The relative influence of boundary conditions and equation simplifications is discussed. This paper presents an explicit, computer solved, mathematical model that is capable of reproducing the overland flow hydrograph.

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## INTRODUCTION

An accurate description of the time distribution of surface runoff is required for many engineering designs. This design information is portrayed graphically as a continuous plot of the instantaneous flow rate with time and is referred to as a discharge hydrograph. Current procedures to synthetically derive this discharge hydrograph range from modifications of the simple Rational Formula to the rather detailed Stanford Model.

Runoff from a watershed due to storm rainfall is affected by the physiographic characteristics of the area as well as the hydrometeorologic characteristics of the rainfall. Commenting on attempts to estimate the hydrograph, Horton (13) states "that success in this area is dependent on a rational understanding of the processes involved in surface runoff, the factors or variables that control these processes, and the relationship between them".

Estimating the performance of a system as complicated as a watershed has resulted in numerous proposed methods. A comprehensive review of the various proposed methods to analyse the watershed as a system has been undertaken by Amorocho and Hart (3) and Liggett and Woolhiser (16). The diversity of these techniques are not as great as first appears. Each work falls into one of two general categories. In one the kinematics of flow alone are considered,



while the dynamic relationships are ignored. In the other category, both the kinematics and dynamics of the flow are taken into consideration. The latter category is the one of interest in this paper and is generally referred to as hydraulic routing.

The theory of spatially varied unsteady flow, i.e., hydraulic routing, has been applied to problems in flood routing (2, 7, 14, 22, 10), surface irrigation (12) and more recently, to describe overland flow (15, 18, 5, 11, 6, 20). This technique is based upon the principles of conservation of mass and Newton's second law of motion. The basic differential equations, as recorded by many writers (25, 4), were developed from these principles by de Saint-Venant in 1871. A solution of these equations has taken either one of two possible avenues of approach. One method has been to simplify the equations by assuming certain terms either negligible or nonexistent. Thus the equations are reduced to a form that can be handled analytically.

More recent methods approach the solution of the complete equations of momentum and continuity by numerical techniques. The basic numerical approach to the solution of this set of equations may be accomplished by either of two accepted techniques: 1) The method of characteristics; or 2) the method of fixed grid intervals employing either the implicit or the explicit technique.

The method of characteristics provides values of the velocity and depth along the characteristic lines. Thus both the time and space interval change in magnitude as the solution proceeds downslope. However, if it becomes necessary to predict or determine the velocity and depth at a given time and location in space, the method can rapidly become unmanageable. Nevertheless, the method can be used if one employs an interpolation technique to find the values of velocity and depth at the desired time and location. Interpolation, using the simplest of techniques, is time consuming, to say nothing of the loss of accuracy. For a complete discussion on the use of the characteristic method for the solution to this type of problem, the reader is referred to (16, 14, 8, 17).

The method of fixed grid intervals differs from the method of characteristics in that it will predict or determine the velocity and depth at equally spaced points in space. The two approaches used in applying this method, a) implicit and b) explicit, differ only in their approach to the solution. In the implicit method, as described by (2), the solution proceeds by solving, at a row, a set of simultaneous equations that describe the flow. The term explicit, as described by (16), refers to a solution that advances point by point from one time line to the next. The difficulty with the explicit methods is one of obtaining a stable solution under varying conditions.

Present stability criteria are not sufficient to insure stability of the explicit technique. The need to develop a better understanding of this basic problem is apparent to the researchers responsible for developing large watershed simulation models (21). The basic component of a large simulation model is the overland flow hydrograph which occurs as an input into the larger main channels. Behavior of an entire simulation model can be altered by underestimating or overestimating the overland flow hydrograph. This overland hydrograph can be described by the spatially varied unsteady flow equations and thus is amenable to solution by numerical techniques (20).

Since a mathematical model based upon the continuity and momentum equations will allow the determination of the quantitative effects of simplification in either the equations or the boundary and initial conditions for which they are solved, the purpose of this paper is as follows:

1. To develop a relationship for the temporal increment,  $\Delta t$ , that will insure convergence and stability of the explicit solution.
2. To determine the quantitative effect of various simplifications used in the explicit solution of the equations of spatially varied unsteady flow.
3. To study the effect of various assumed boundary and initial conditions on the explicit solution to the spatially varied unsteady flow equations.
4. To study the effect of subarea shape on the temporal distribution of runoff of overland flow.

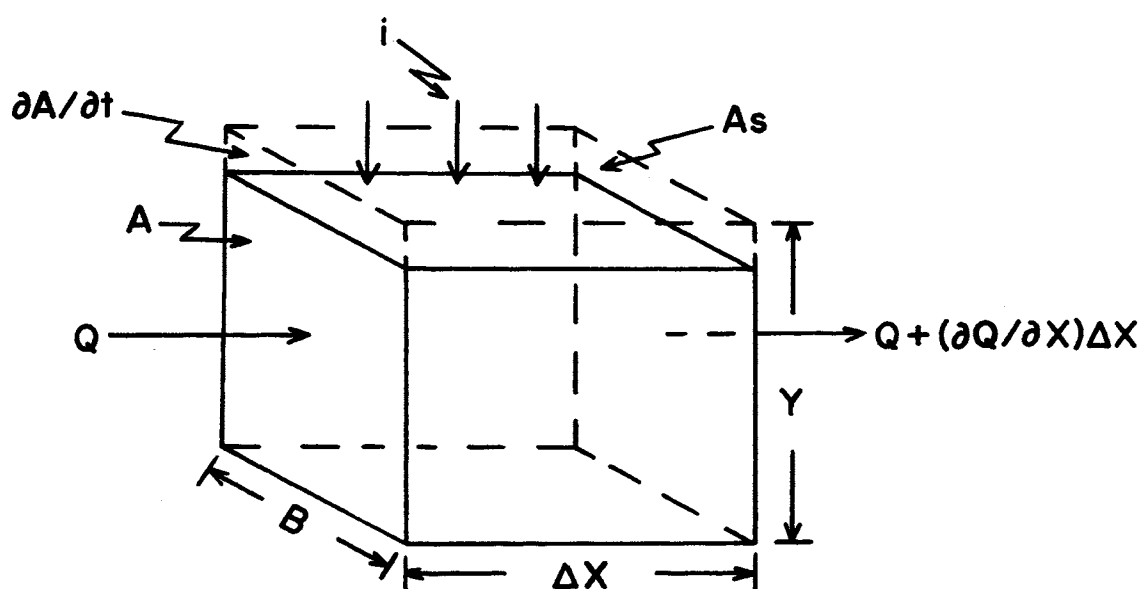
5. To develop criteria for an explicit numerical solution of a mathematical model that would allow simulation of the overland flow hydrograph for small watersheds.

#### DEVELOPMENT OF THE BASIC MATHEMATICAL MODEL

The direct runoff from an overland flow plane is effected by the internal processes occurring within the boundaries of this plane. These internal processes are very complex and for most situations are nonlinear. The temporal distribution of the rainfall excess, i.e., the discharge hydrograph, is a result of the action of a distributed dynamic system (11). The behavior of this system can be expressed by a pair of hyperbolic partial differential equations known as the spatially varied unsteady flow equations. These equations can be developed from the basic concept of conservation of mass and the momentum principle given by Newton's second law of motion. The equation of continuity will account for the conservation of mass, and Newton's second law of motion will describe the dynamic behavior of the system. The validity of these two equations in describing overland flow has been established by many investigators (15, 19, 5, 20).

*Continuity of Flow.* - The equation of continuity simply states that the inflow minus the outflow is equal to the change in storage within the system. In order to relate this to the overland flow plane, consider an element as shown in figure 1.

FIGURE 1. TYPICAL ELEMENT FOR DERIVATION OF  
THE CONTINUITY EQUATION



From continuity

$$Q(\text{in}) - Q(\text{out}) = Q(\text{stored}) \dots \dots \dots (1)$$

From figure 1

$$Q(\text{in}) = Q + ikA_s \dots \dots \dots (2)$$

$$Q(\text{out}) = Q + (\partial Q / \partial X) \Delta X \dots \dots \dots (3)$$

$$Q(\text{stored}) = (\partial A / \partial t) \Delta X \dots \dots \dots (4)$$

Substitution of equations 2, 3, and 4 into equation 1 and simplifying results in the general continuity equation for an arbitrary width of overland flow plane.

$$ikA_s = (\partial Q / \partial X) \Delta X + (\partial A / \partial t) \Delta X \dots \dots \dots (5)$$

where the conversion factor  $k$  transforms the rainfall excess intensity in inches per hour to feet per second,  $A_s = B \Delta X$ , is the surface area of the element and  $A = BY$  is the cross sectional area of flow. Remembering that  $Q = AV$ , the partial derivative of  $Q$  with respect to  $X$  becomes  $\partial(AV) / \partial X$  which is equal to  $A(\partial V / \partial X) + V(\partial A / \partial X)$ . The variables  $B$ ,  $Y$ , and  $V$  are functions of both  $X$  and  $t$ ; thus,  $\partial A / \partial X$  equals  $\partial(BY) / \partial X$  which results in  $B(\partial Y / \partial X) + Y(\partial B / \partial X)$  and the  $\partial A / \partial t$  becomes  $\partial(BY) / \partial t$  which equals  $B(\partial Y / \partial t) + Y(\partial B / \partial t)$ . Substitution of these relationships into equation 5 and simplifying produces

$$ikB = BY(\partial V / \partial X) + V(B(\partial Y / \partial X) + Y(\partial B / \partial X)) + B(\partial Y / \partial t) + Y(\partial B / \partial t) \dots \dots \dots (6)$$

This is the general form of the continuity equation and will be used in developing the proposed mathematical model. Equation 6 reduces to the form shown by Morgali (20) and Harbaugh (11), for a unit width.

*Momentum Considerations of Flow.* - Newton's second law of motion states that the sum of all external forces acting on a body must be equal to the change of momentum of the body with respect to time. The derivation of the momentum equation is based on the following assumptions:

1. The fluid is incompressible.
2. The conditions of flow are such that only hydrostatic pressure exists.
3. The momentum correction factor is equal to unity.
4. The kinetic energy correction factor is one.
5. The overland flow plane slope is small; therefore,  $\sin \theta$  is equal to tangent  $\theta$ , i.e.  $\tan \theta = S_o$ .
6.  $V$  is the average velocity across the section.
7. Momentum of the rainfall has been neglected.
8. The slope of the energy grade line can be approximated by the Chezy-Manning relationship for steady flow.

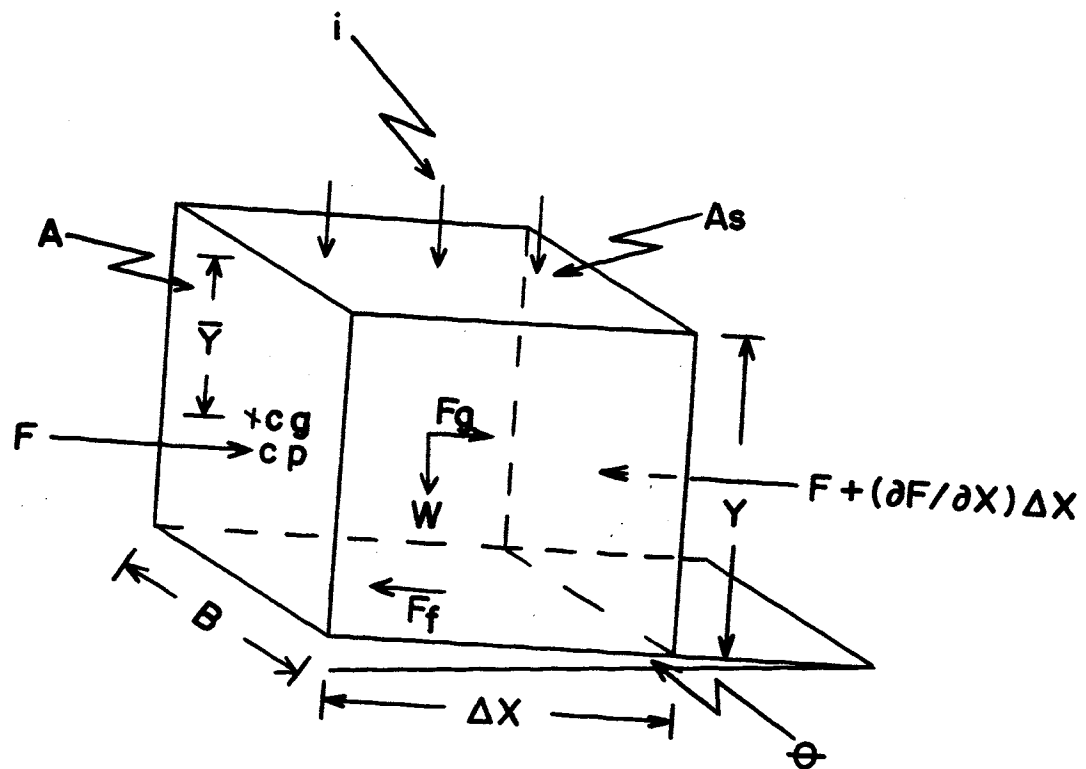
Newton's second law of motion can be expressed as

$$\Sigma(\text{Forces}) = D(MV)/Dt \dots \dots \dots (7)$$

Summing the external forces shown in figure 2, results in



FIGURE 2. TYPICAL ELEMENT FOR DERIVATION OF  
THE MOMENTUM EQUATION



$$\Sigma(\text{Forces}) = \gamma A \Delta X (S_o - S_f) - \gamma \Delta X (\bar{Y} (B (\partial Y / \partial X) + Y (\partial B / \partial X)) + BY (\partial \bar{Y} / \partial X)) \dots \dots \dots (8)$$

The rate of change of momentum can be expressed as  $M(DV/Dt) + V(DM/Dt)$  where  $DM/Dt$  is the addition of mass with time resulting from the incoming rainfall. For the case of a constant intensity and spatial distribution of rainfall,  $DM/Dt$  reduces to  $pikA_s$ . Remembering that  $V$  is a function of both  $X$  and  $t$ , the total derivative is  $DV/Dt = V(\partial V / \partial X) + \partial V / \partial t$ . Equating the sum of the external forces to the rate of change in momentum results in

$$Ag(S_o - S_f) = g(B\bar{Y}(\partial Y / \partial X) + Y\bar{Y}(\partial B / \partial X) + BY(\partial \bar{Y} / \partial X)) + A(V(\partial V / \partial X) + \partial V / \partial t) + V(ikB) \dots \dots \dots (9)$$

The cross sectional area of overland flow is rectangular; therefore,  $A = BY$  and  $\bar{Y} = Y/2$ . Substitution of these relationships into equation 9 produces

$$gB(S_o - S_f) = g(B(\partial Y / \partial X) + (Y/2)(\partial B / \partial X)) + B(V(\partial V / \partial X) + \partial V / \partial t) + (V/Y)(ikB) \dots (10)$$

which is the momentum equation to be used in developing the proposed mathematical model. Equation 10 reduces to the equation used by Morgali (20) and Harbaugh (11) for a unit width section.

## NUMERICAL SOLUTION OF THE MATHEMATICAL MODEL

Equations 6 and 10 form a pair of quasi-linear hyperbolic partial differential equations (14, 18, 22). For the overland flow plane, the solution to this set of equations can be described as an initial value problem (see figure 3). The solution of this system of equations may be accomplished by numerous numerical techniques. The explicit technique of the method of fixed grid intervals will be the approach used in this paper. Crandall (9) describes this type of solution as "marching out a solution on an open boundary which is bounded by known conditions". The explicit numerical technique is applicable to overland flow, since both the initial and the boundary conditions are known.

For the overland flow plane situation, i.e., a rectangular cross sectional area, equations 6 and 10 reduce respectively to equations 11 and 12.

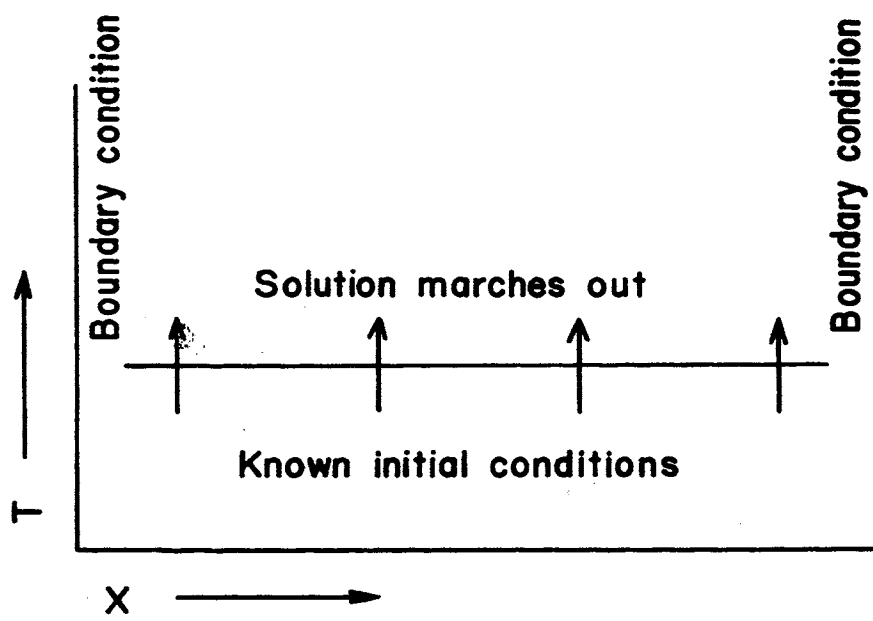
(continuity)

$$ik = Y(\partial V / \partial X) + V((\partial Y / \partial X) + (Y/B)(\partial B / \partial X)) + \partial Y / \partial t \dots \dots \dots (11)$$

(momentum)

$$g(S_o - S_f) = g((\partial Y / \partial X) + (Y/2B)(\partial B / \partial X)) + V(\partial V / \partial X) + (\partial V / \partial t) + (V/Y)ik \dots \dots \dots (12)$$

FIGURE 3. DESCRIPTION OF INITIAL VALUE PROBLEM



Equations 11 and 12 are used to describe either the sub-critical or the supercritical overland flow regime.

*Central Differences.* - The partial derivatives in equations 11 and 12 can be expressed by using a finite difference scheme known as central differences. A portion of the x-t plane showing the grid system for this technique is found in figure 4. Substituting the central differences shown in figure 4 into equation 11 and rearranging produces

$$Y_P = Y_M + \Delta t(Y_M((V_L - V_R)/2\Delta X) + V_M((Y_L - Y_R)/2\Delta X) + ((V_M Y_M)/B_M)((B_L - B_R)/2\Delta X) + ik)....(13)$$

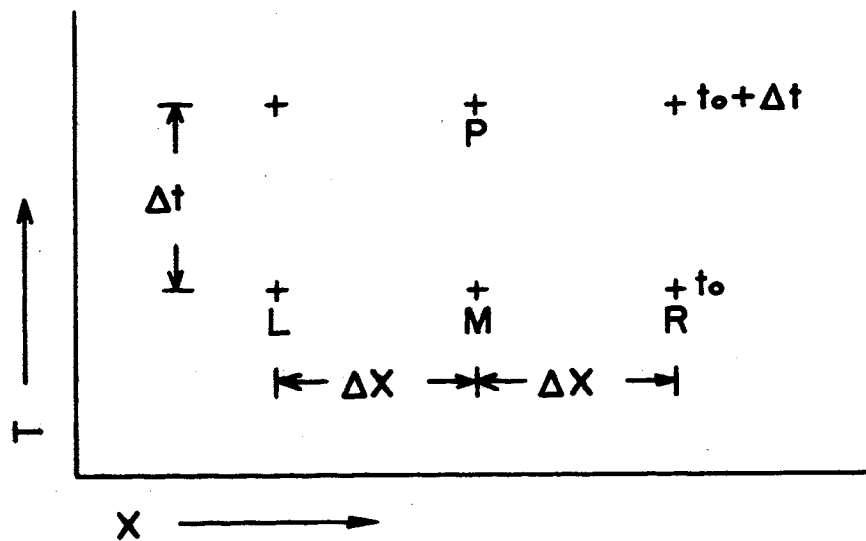
If the velocity and depth are known at time  $t_0$  the only unknown in equation 13 is  $Y_P$ , the depth of flow at time  $t_0 + \Delta t$ . Solution of equation 13 for all known values of  $V$  and  $Y$  at time  $t_0$  will produce values of  $Y_P$  at the new time  $t_0 + \Delta t$ . Thus by knowing the initial values of  $V$  and  $Y$ , the solution may be marched forward  $\Delta t$  in time.

Substitution of the central differences noted in figure 4 into equation 12 results in

$$g(S_0 - S_f) = g(((Y_R - Y_L)/2\Delta X) + (Y_M/2B_M)((B_R - B_L)/2\Delta X)) + V_M(((V_R - V_L)/2\Delta X) + (V_P - V_M)/\Delta t) + (V_P/Y_P)ik...(14)$$

FIGURE 4. CENTRAL DIFFERENCE FINITE DIFFERENCE NETWORK





$$\partial V / \partial X)_M = (V_R - V_L) / 2\Delta X \quad \partial V / \partial t)_P = (V_P - V_M) / \Delta t$$

$$\partial Y / \partial X)_M = (Y_R - Y_L) / 2\Delta X \quad \partial Y / \partial t)_P = (Y_P - Y_M) / \Delta t$$

$$\partial B / \partial X)_M = (B_R - B_L) / 2\Delta X \quad \partial B / \partial t)_P = (B_P - B_M) / \Delta t$$

All values in equation 14 are known at time  $t_0$  except  $S_f$ ; and at time  $t_0 + \Delta t$ , the only unknown is  $V_P$ , since  $Y_P$  has been obtained in equation 13. At this stage in the solution of the equations it becomes necessary to evaluate the friction slope term,  $S_f$ . This term is one of the more important terms in the dynamic equation. The magnitude of this term is usually larger than any of the other terms except  $S_0$ ; therefore, the method of evaluating  $S_f$  would be expected to have a large effect on the solution of the mathematical model.

Although the Chezy-Manning equation was developed for steady uniform flow, it has been shown to give reasonable results for unsteady nonuniform flow if the unsteady changes are gradual (16, 22, 11, 4, 14). The Chezy-Manning equation may be expressed as

$$S_f = V^2 N^2 / 2.2082 R^{4/3} \dots \dots \dots (15)$$

where  $N$ , the roughest coefficient, is to be assumed constant. For the overland flow plane  $B \gg Y$ , therefore, as shown by Chow (7),  $R \approx Y$  which reduces equation 15 to

$$S_f = V^2 N^2 / 2.2082 Y^{4/3} \dots \dots \dots (16)$$

If  $S_f$  is evaluated at time  $= t_0 + \Delta t$ , at station  $P$ , as several writers have indicated, (18, 22, 11, 5) then equation 16 becomes

$$S_f)_P = V_P^2 N^2 / 2.2082 Y_P^{4/3} \dots\dots\dots (17)$$

Substituting equation 17 into equation 14 and rearranging produces an equation that is quadratic in  $V_P$ . Solving this equation by completing the square produces

$$V_P = -C + (C^2 + C1 (gS_o + V_M((1/\Delta t) + (V_L - V_R) / 2\Delta X) + g((Y_L - Y_R)/2\Delta X) + (Y_M/2B_M)(B_L - B_R) / 2\Delta X)))^{1/2} \dots\dots\dots (18)$$

where

$$C = (2.2082 Y_P^{4/3})(Y_P + \Delta t i_k) / 2gN^2 \Delta t Y_P$$

$$C1 = 2.2082 Y_P^{4/3} / gN^2$$

If the values for velocity and depth are known at time  $= t_o$ , the only unknown in equation 18 is  $V_P$ , the velocity at time  $t_o + \Delta t$ , since the value for  $Y_P$  has been obtained by the solution of equation 13. Thus, if equation 13 is solved first for  $Y_P$ , equation 18 can readily be solved for  $V_P$ . This allows the complete solution for  $V$  and  $Y$  to march forward an increment of time,  $\Delta t$ , for a particular station. The solution of equations 13 and 18 are undertaken for each succeeding station down the overland flow plane at time  $t_o + \Delta t$ . For the next time increment, at this station, the values of  $V_P$  and  $Y_P$  at time  $t_o + \Delta t$  for each station become  $V_M$  and  $Y_M$  for time  $t_o + 2\Delta t$

at the respective station. The solution is thus "marched out" to time  $t_0 + n\Delta t$  through the successive application of equations 13 and 18.

*Equation Simplifications.* - Examination of the literature reveals that equation 13 differs somewhat from that presented by Harbaugh, Morgali, and others (11, 20, 24). The difference is a result of the averaging assumption used to evaluate  $Y_M$  and  $V_M$ . Their technique used the relationship

$$V_M = (V_L + V_R)/2$$

$$Y_M = (Y_L + Y_R)/2$$

Substituting these relationships into equation 13 results in

$$Y_P = Y_M + \Delta t(((V_L Y_L - V_R Y_R)/2\Delta X) + (V_M Y_M/B_M) \\ ((B_L - B_R)/2\Delta X) + ik) \dots \dots \dots (19)$$

Other modifications of the equations can be obtained by evaluating  $S_f$  at the station M rather than P. This approach results in a momentum equation of the form

$$V_P = ((\Delta t Y_P)/(Y_P + \Delta t ik))(gS_0 - ((V_M^2 N^2 g)/ \\ 2.2082 Y_M^{4/3}) + g(((Y_L - Y_R)/2\Delta X) + (Y_M/2BM) \\ ((B_L - B_R)/2\Delta X)) + V_M((1/\Delta t) + ((V_L - V_R) \\ 2\Delta X))) \dots \dots \dots (20)$$

*Initial Conditions.* - At a given time all velocities and depths of flow must be known along the network to start the numerical solution, i.e., the initial values must be given or readily calculated. For the initially dry condition, which is the usual case for overland flow, all velocities and depths are equal to zero at time  $t_0$ , (the time rainfall begins). However, if the overland flow plane is not dry, it becomes necessary to determine the velocity and depth of flow at all stations at time  $t_0$  before the solution can proceed. The overland flow plane is assumed to be dry at the beginning of rainfall for all tests reported in this paper. Therefore, the initial values of the velocity and depth at all stations is zero.

*Upstream Boundary Conditions.* - The flow at the beginning of the overland flow plane is assumed to be zero. Therefore, all water on the plane will enter as rainfall and the velocity at station 1, the upstream end, will be zero and will remain such for all time. The depth of flow at station 1, however, will vary according to whether the flow is supercritical or subcritical. Four separate treatments of the upstream boundary condition will be shown to demonstrate the variety of boundary conditions used in solving the explicit numerical solution.

UP1. The depth of flow at station 1 will remain zero for all time. This condition would apply if the flow regime is found or assumed to be supercritical.

UP2. - The flow profile is assumed to be such that the depths at stations 1 and 2 are equal. This condition would be applicable if the flow in section 1-2 was uniform.

UP3. - The flow regime is assumed such that the backwater effect can be accounted for by the continuity equation. The grid system used in this technique is shown in figure 5. Substituting the finite differences shown in figure 5 into equation 11 and rearranging, remembering that  $V_M = 0.0$ , gives

$$Y_P = Y_M + \Delta t(ik - Y_M V_R / \Delta X) \dots \dots \dots (21)$$

UP4. - The depth at station 1 will be allowed to change as the storage in section 1-2 changes. The storage relationship can be developed from the continuity equation which states that the change in storage between stations 1 and 2, over a given time interval is equal to the average flow into the section minus the average flow out of the section. Expressed in equation form

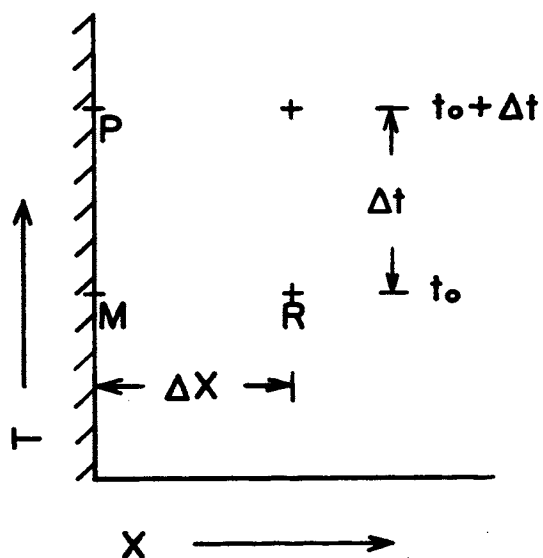
$$\text{Storage} = \Delta t((I_1 + I_2)/2 - (O_1 + O_2)/2) + \text{Storage} \\ @ t + \Delta t \quad \quad \quad @ t$$

Making the proper substitutions for inflow,  $I$ , outflow,  $O$ , and storage in this equation and solving for  $Y_P(1)$ , the depth at station 1 at time  $= t_0 + \Delta t$  gives

$$Y_P(1) = Y(1) + Y(2) - Y_P(2) + \Delta t (RA1 + RA2 - \\ (B(2)(Y(2) V(2) + Y_P(2) VP(2))/AS)) \dots \dots (22)$$

where  $AS = \Delta X(B(1) + B(2))/2$ ,  $RA1$  and  $RA2$  equals  $ik$  or the rainfall at time  $t_0$  and time  $t_0 + \Delta t$ , respectively. This relationship would only apply if the flow regime is subcritical.

FIGURE 5. BACKWARD DIFFERENCE FINITE  
DIFFERENCE NETWORK



$$\partial V / \partial X)_M = (V_R - V_M) / \Delta X \quad \partial V / \partial t)_P = (V_P - V_M) / \Delta t$$

$$\partial Y / \partial X)_M = (Y_R - Y_M) / \Delta X \quad \partial Y / \partial t)_P = (Y_P - Y_M) / \Delta t$$

$$\partial B / \partial X)_M = (B_R - B_M) / \Delta X \quad \partial B / \partial t)_P = (B_P - B_M) / \Delta t$$



*Downstream Boundary Conditions.* - The downstream boundary conditions encountered in overland flow depend upon the flow regime along the flow plane and the outlet conditions. The state of flow at the boundary can be considered to be critical or noncritical. A critical condition would exist if the boundary was located at a point where critical flow existed or where a free overfall situation occurred and the flow upstream was subcritical. The possibility of maintaining critical flow all along the plane is rather remote; therefore, consideration of this state of flow will not be undertaken. A noncritical condition would exist if the boundary is located at a point where the flow is noncritical or where the flow does not pass through critical depth. Four separate treatments of the downstream boundary condition will be investigated to demonstrate the variety of downstream boundary methods used in solving the explicit numerical solution.

DN1. The depth and velocity at the downstream station M are assumed equal to the depth and velocity at station M-1. This condition would be true only if the flow between stations M and M-1 was uniform.

DN2. - The plane is assumed to be extended to a fictitious station (M+1) and the depth and velocity at station M+1 is assumed equal to those at station M. This technique could be used for all noncritical boundary condition. An advantage of this over other techniques is that the depth and velocity at all stations along the plane can be calculated by central differences.

DN3. - The depth and velocity at station M is calculated using the continuity and momentum equations.

The forward difference scheme used to determine the finite differences for this downstream boundary condition is shown in figure 6. Substituting the finite differences shown in figure 6 into equation 11 and solving for  $Y_P$  results in

$$Y_P = Y_M + \Delta t(ik + Y_M((V_L - V_M)/\Delta X) + V_M(((Y_L - Y_M)/\Delta X) + (Y_M/B_M)((B_L - B_M)/\Delta X))) \dots \dots \dots (23)$$

Similarly, substituting these forward differences into equation 12 and evaluating  $S_f$  by equation 17

$$V_P = -C + (C^2 + C1(gS_o + V_M((1/\Delta t) + ((V_L - V_M)/\Delta X)) + g(((Y_L - Y_M)/\Delta X) + (Y_M/2B_M)((B_L - B_M)/\Delta X))))^{1/2} \dots \dots \dots (24)$$

where C and C1 are as shown on page 20.

This condition, as well as condition 2, is applicable for all noncritical states of flow.

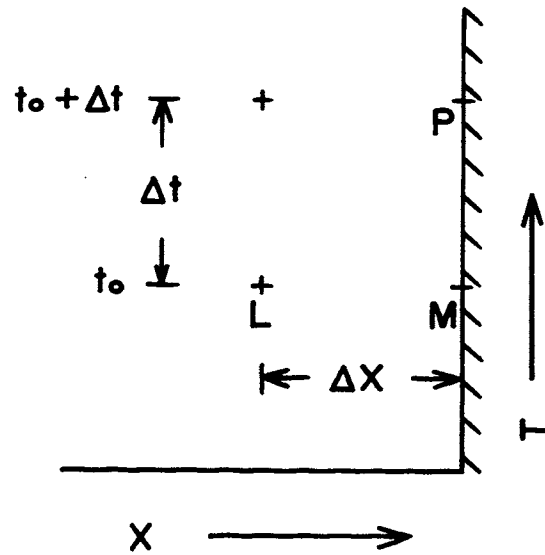
DN4. - The flow is assumed to be such that the depth at station M is critical. This depth can then be determined from an equation of the form  $Y_c = (Q^2/B^2g)^{1/2}$  where Q is the flowrate at station M. This flow can be obtained from the known flow at station M-1 and the added flow, due to rainfall, between station M-1 and M. With the depth at station M known, the velocity can be found from the relationship  $V = (gy)^{1/2}$ . This condition would apply in the free overfall situation, provided the flow upstream is subcritical.

#### STABILITY AND CONVERGENCE

Not all numerical schemes used to approximate differential equations converge as  $\Delta t$  and  $\Delta X$  approach zero.

Stability is a property that guarantees convergence to a solution when employing a difference scheme. In general

FIGURE 6. FORWARD DIFFERENCE FINITE  
DIFFERENCE NETWORK



$$\partial V / \partial X)_M = (V_M - V_L) / \Delta X \quad \partial V / \partial t)_P = (V_P - V_M) / \Delta t$$

$$\partial Y / \partial X)_M = (Y_M - Y_L) / \Delta X \quad \partial Y / \partial t)_P = (Y_P - Y_M) / \Delta t$$

$$\partial B / \partial X)_M = (B_M - B_L) / \Delta X \quad \partial B / \partial t)_P = (B_P - B_M) / \Delta t$$

stability deals with the boundedness of the solution of the difference equation. In an unstable scheme, small disturbances grow without bound and soon mask the desired solution. The criteria used to define acceptable stability and convergence for the mathematical model of overland flow is defined as one in which the rising limb of the overland hydrograph converges uniformly to equilibrium flow.

Stoker (23), Liggett (16), and others state that they encountered no stability problems with the method of characteristics. Amein (2), Brutsaert (4), and Liggett (16) report similar experiences with the implicit technique used with the method of fixed grid intervals. However, Wylie (24) states that convergence problems are encountered if extreme values are utilized when selecting  $\Delta t$  and  $\Delta X$  increments.

Harbaugh (11), Morgali (20, 19), Keulegan (15), and other investigators have recognized the Courant condition  $\Delta X / \Delta t \leq V + (gY)^{1/2}$  as a necessary but not a sufficient condition to insure the stability of the explicit technique used in the method of fixed grid intervals. Perkins (21) states that the explicit fixed grid scheme was found sufficiently stable and convergent for unsteady flow computations if  $(V + (gY)^{1/2})(\Delta t / \Delta X) \leq 1 - ((gN^2 V) / (2.21 Y^{4/3})) \Delta t$  was satisfied. Harbaugh found that to insure stability and convergence for the conditions of

his research that a  $\Delta X = 1.0$  ft. and a  $\Delta t = 0.1$  sec. were necessary. This indicates a  $\Delta X = L/40$  and a  $\Delta t = \Delta t_i/10$  where  $\Delta t_i$  is the value obtained from the Courant condition when applied at equilibrium flow.

Once  $\Delta X$  is chosen, the  $\Delta t$  increment must be limited in such a way that station P (see figure 4) falls within the region bounded by the forward characteristic emanating from station L and the backward characteristic emanating from station R (16, 14, 18). The convergence and stability of the explicit technique depends upon the magnitude of this temporal and spatial increment. At present most methods used to determine the proper magnitude of the temporal increment,  $\Delta t$ , are based upon some modification of the Courant condition. The drawback to the present approaches stems from having to predict the velocity and depth of flow prior to determining  $\Delta t$  and  $\Delta X$ .

From the practical standpoint of trying to simulate natural conditions, there should be a relationship between these temporal and spatial increments and the physical parameters of the overland flow plane. Since the mathematical model will allow the study of the effects of varying various physical parameters, the following relationship is postulated:

$$\Delta t = f(\Delta X, S_o, L, N, \dots) \dots \dots \dots (25)$$

This relationship can further be assumed to follow a

mathematical model of the form

$$\Delta t = C \Delta X^W S_o^X L^Y N^Z \dots \dots \dots (26)$$

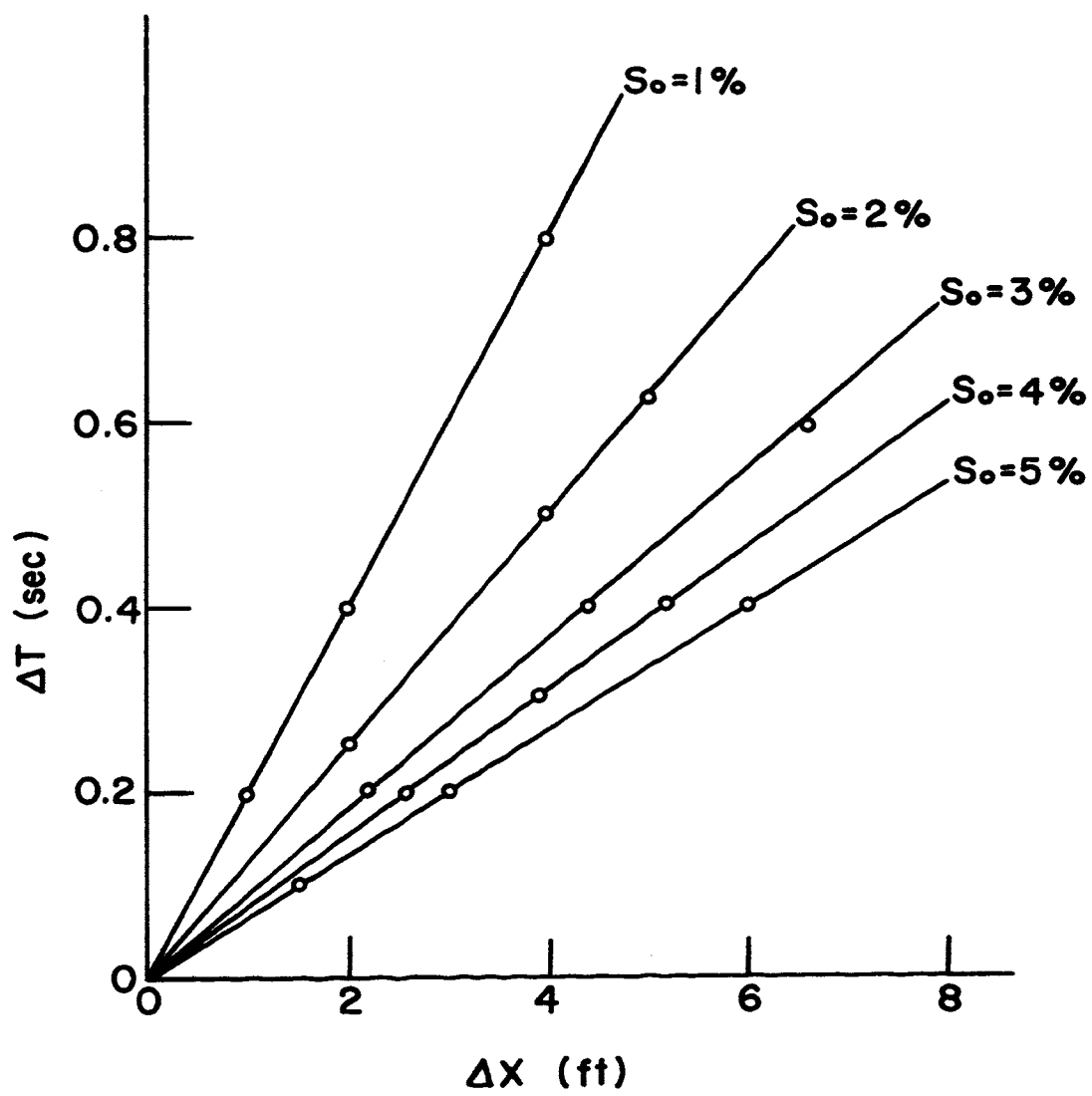
#### TESTING OF THE MATHEMATICAL MODEL

The mathematical model tested was the explicit solution of equations 13 and 18 with zero initial flow and boundary conditions DN2 and UP1. The length of the overland flow plane L varied from 10.0 through 200.0 ft., width B ranged from 1.0 through 100.0 ft., rainfall intensity i increased from 2.16 through 12.96 in. per hr., the slope  $S_o$  varied from 1 to 5%, and Manning's n varied from .01 through .05. Each of these variables caused a distinct change in the rising portion of the hydrograph. For example, as L increased, time to equilibrium increased; thus the rising limb flattened. This is understandable because as L increases, so does the time of concentration.

Once a stable solution was obtained for a particular value of  $\Delta t$  and  $\Delta X$ , ranges of L and n within the limits previously defined did not effect stability or convergence. However, changing  $S_o$  from 1% to 5% caused the solution to become unstable. This is understandable since Wylie (24) states that stability of the explicit fixed grid solution is dependent on frictional losses. Increasing  $S_o$  would thus cause increased frictional losses which would reduce the allowable  $\Delta t$  increment. Figure 7 shows that as  $S_o$

FIGURE 7. STABILITY LIMITS FOR EXPLICIT  
MATHEMATICAL MODEL





increases, the limiting ratio of  $\Delta X/\Delta t$  increases. Stated another way, as  $S_o$  increases,  $\Delta t$  decreases for a constant  $\Delta X$ . Applying a linear regression technique to these data shown in figure 7 produced equation 27.

$$\Delta t = 0.36 X^{0.48} S_o^{-0.58} L^0 N^0 \dots \dots \dots (27)$$

where  $\Delta t$  is in seconds,  $\Delta X$  in feet,  $S_o$  in %,  $L$  in feet, and  $N$  is Manning's "n". This equation may be simplified to

$$\Delta t = 0.2 X/S_o \dots \dots \dots (28)$$

Use of the simplified equation will result in a slightly reduced value for  $\Delta t$ ; however, the additional computer time necessary with the shorter  $\Delta t$  is insignificant for the range of variables tested.

*Boundary Conditions.* - To test quantitatively the various boundary conditions DN1 through DN4 and UP1 through UP4, the following set of hypothetical data was employed:  $\Delta X = 2.5$  ft.,  $\Delta t = 0.5$  sec.,  $B = 50$  ft.,  $L = 100.0$  ft.,  $N = 0.01$ ,  $S_o = 1.0\%$  and  $i = 6.0$  in/hr. Equilibrium flow for this set of data is 0.694 cfs. The results of these tests are shown in table 1 as models 1 through 7. These models each vary from the originally selected mathematical model by the boundary condition indicated. For example in model 2 the upstream boundary condition UP1 is replaced by UP2. From table 1 it is seen that the original mathematical

TABLE 1. RESULTS OF MODELS TESTED

MODEL NUMBER	MODEL COMPONENTS	PERCENT SURGE	MODEL EQUILIBRIUM DISCHARGE	PERCENT DE- PARTURE FROM EQUILIBRIUM
1	Eqs 13 and 18 DN2, UP1	1.7	0.694	0.0
2	Eqs 13 and 18 DN2, UP2	2.9	0.704	+1.4
3	Eqs 13 and 18 DN2, UP3	2.7	0.703	+1.3
4	Eqs 13 and 18 DN2, UP4	2.5	0.701	+1.0
5	Eqs 13 and 18 DN1, UP1	Did not reach Equilibrium	0.677	-2.5
6	Eqs 13 and 18 DN3, UP1	0.6	0.685	-1.3
7	Eqs 13 and 18 DN4, UP1	3.7	Unstable	
8	Eqs 19 and 18 DN2, UP1	3.2	0.703	+1.3
9	Eqs 13 and 20 DN2, UP1	Solution very unstable		

was the only one to converge to equilibrium flow. Model 5 shows that the assumption of uniform flow between stations M-1 and M is not applicable, since this mathematical model did not reach equilibrium flow. Model 6 shows the error associated with the difference scheme used for this downstream boundary. This error is of the order of magnitude  $h$  where  $h = \Delta X/L$ . Models 8 and 9 in table 1 were formulated using the simplified equations 19 and 20. Model 8 as shown in table 1 did not converge to equilibrium flow since the assumed linear variations in  $Y$  and  $V$  as shown in equation 19 will not hold for spatially varied unsteady flow.

A close examination of table 1 would show that combining models 6 and 8 in the form of equations 19 and 18 with UP1 and DN3 should converge to equilibrium flow. This combination was used by Harbaugh (11) and Morgali (18) and found to converge to equilibrium flow for a similar range of variables employed in this study; although models 6 and 8 individually produce either an underestimation or overestimation of equilibrium flow by 1.3%.

The use of model 9 produces a very unstable solution for an initially dry overland flow plane. When used in an overland flow plane with a significant amount of existing flow, the equation will become stable. This is apparent by noting

that in equation 20, the initial value of  $Y_M$  for a dry overland flow plane equals zero, and division by  $Y_M$  gives an answer of infinity for certain terms in the finite difference form of the momentum equation.

*Existing Stability Criteria.* -- Testing of the mathematical model revealed that the time to equilibrium was influenced by  $\Delta X$  and  $\Delta t$ . Tables 2 and 3 show a portion of the results. The conditions of flow used are:  $L = 100$  ft.,  $B = 50$  ft.,  $S_o = 1\%$ ,  $N = .01$ ; and  $i = 6.0$  in/hr. These conditions produce an equilibrium flow of 0.694 cfs with a velocity of 0.9004 fps and a depth of 0.0154 ft. The Courant equation for these conditions gives:

$$\Delta t \leq \frac{\Delta X}{1.6} \leq 1.5 \text{ sec.}$$

The Perkins equation gives:

$$\Delta t \leq \frac{0.00862 \Delta X}{0.0138 + 0.0029 \Delta X} \leq 1.02 \text{ sec.}$$

The results of the convergence criteria developed in equation 28 results in

$$\Delta t \leq 0.2 \Delta X / S_o \leq 0.5$$

As noted in table 2, the solution is unstable for  $\Delta t = 1.0$  sec. Therefore, neither the Courant nor the Perkins criteria is satisfactory for this particular choice of physical parameters of the overland flow plane.

TABLE 2. EFFECT OF VARYING  $\Delta t$  WITH  $\Delta X = L/40$ 

$\Delta t$ Second	$\frac{\Delta X}{\Delta t}$	Time to Equilibrium Second	Percent Surge
0.1	25	117	0.4
0.2	12.5	115	0.6
0.5	5	113	2.1
1.0	2.5	Unstable	Unstable

TABLE 3. EFFECT OF VARYING  $\Delta t$  WITH  $\Delta X = L/10$ 

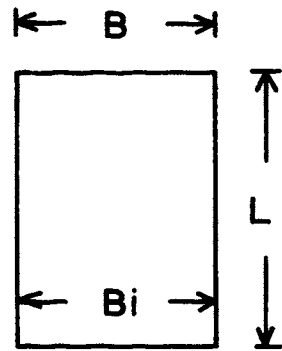
$\Delta t$ Second	$\frac{\Delta X}{\Delta t}$	Time to Equilibrium Second	Percent Surge
0.1	100	126	1.1
0.2	50	125	1.2
0.5	20	123	1.4
2.0	5	117	3.4
5.0	2	112	Unstable

A variation of  $\Delta X$  in the mathematical model is shown in table 3. The Courant equation results in a  $\Delta t \leq 6.25$  sec., the Perkins criteria produces a  $\Delta t \leq 2.02$  sec., and equation 28 results in a  $\Delta t \leq 2.0$  seconds. Table 3 shows the solution is stable for the Perkins criteria and equation 28 but unstable for the Courant condition. Tables 2 and 3 show, as did Morgali (18), that holding  $\Delta X$  constant and increasing  $\Delta t$  will decrease the time to equilibrium flow and increase the surge. It should be noted that holding  $\Delta t$  constant and increasing  $\Delta X$  will increase the time to equilibrium flow. The surge increases but not as rapidly as before.

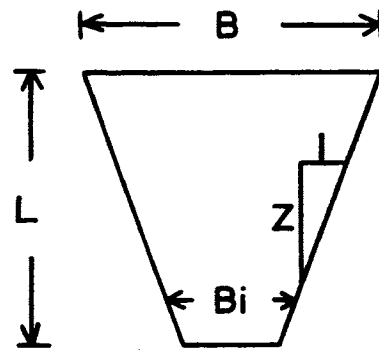
*Subarea Shape.* - At present most methods used do not permit one to consider the shape of the subareas into which a simulation model of the watershed may be divided. Most attempts are made by assuming unit width strips, thus only a rectangular subarea exists. To study the effect that the shape of a subarea has on the overland flow hydrograph, the shapes shown in figure 8 have been considered. The explicit mathematical model is capable of solving these and other configurations because it permits  $B$  to vary in space as well as time. For the overland flow simulation, the cross sectional area of flow is assumed to be rectangular; therefore, the change in  $B$  with time is zero. Thus, it is necessary only to find the relationship  $B = f(x)$ . This relationship, for the areas

FIGURE 8. SUBAREA SHAPES

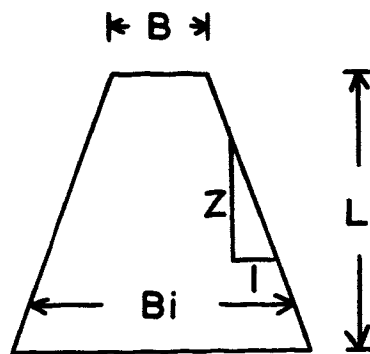




Rectangular



Trapezoidal



Inverted trapezoidal

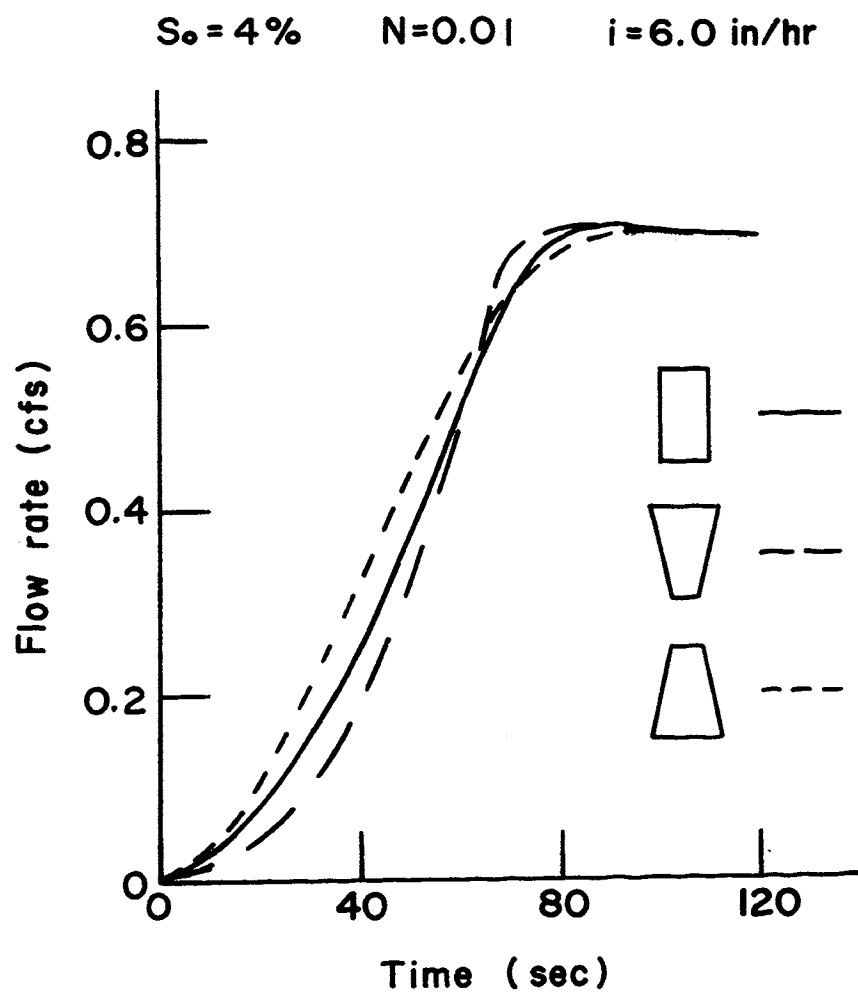
shown in figure 8, is  $B_i = B \pm 2 (BM) (i - 1) \Delta X$ , where  $B$  is the initial width and  $i$  represents the stations along the plane spaced  $\Delta X$  feet apart.  $BM$  is shown on the figure as a measure of the slope of the trapezoidal shaped area.

The results of tests performed on the shapes indicated in figure 8 are shown in figure 9. Each shape had an area equal to 5000 sq. ft. and a length of 100 ft. The slope for the trapezoidal shapes equal to 1:4. These results show that the inverted trapezoidal shaped area will produce the higher initial flowrate. This phenomena can be explained by realizing that the initial area contributing to flow is larger for this shape. The trapezoidal shape demonstrates a larger change of flow with time as time approaches the time to equilibrium. This is a result of the larger amount of available detention storage on the upper portions of the subarea as the time to equilibrium is approached. The order in which the various shapes reached equilibrium flow is first, trapezoidal; second, rectangular; and third, inverted trapezoidal.

#### SUMMARY AND CONCLUSIONS

The flow conditions used in this study were such that after approximately 20 seconds the entire flow became supercritical. If the flow regime is such that

FIGURE 9. RESULTS OF TESTS PERFORMED ON  
SUBAREA SHAPES



subcritical flow does exist; then UP2, 3, or 4 could be used in the mathematical model. Tests were made using UP2, 3, and 4 which revealed that the upstream boundary assumption had little, if any, effect upon the outflow hydrograph. However, each boundary condition did produce a slightly different surface profile for the initial portion of the overland flow plane. After a few iterations of time, the profiles converged into one at a distance equal to approximately five-tenths the overland flow plane length. Thus, for the range of variables investigated, the choice of the upstream boundary condition could always be assumed as UP1.

This study showed DN2 to be the best of those downstream boundary conditions used to describe a noncritical downstream boundary. DN2 was tried and found to be satisfactory for either sub or supercritical flow. However, if the boundary conditions are such that a free overfall exists and the flow regime upstream is subcritical, DN4 can and should be used.

The principal discovery in this report is the fact that  $\Delta t$  is a function of known physical features of each subarea. Therefore, it is no longer necessary to find by trial, or by predicting  $V$  and  $Y$  at equilibrium, a  $\Delta t$  that will insure convergence and stability. The proper value of  $\Delta t$  can be obtained from equation 28 by selecting

a suitable value of  $\Delta X$  and determining the overland flow plane slope. A suitable value of  $\Delta X$  will be obtained by letting  $\Delta X$  have a value within the range of 0.025 L to 0.1 L.

The explicit mathematical model developed in this study is capable of simulating the overland flow hydrograph. This model has the following advantages:

1. An uncomplicated explicit numerical solution technique which involves only a basic algebra background.
2. The ease with which the  $\Delta X/\Delta t$  ratio may be evaluated, which will insure stability and convergence.
3. The simplicity of changing boundary conditions and evaluating the relative error.
4. A general equation formulation suitable for simulating various subarea shapes.

The equations developed and conclusions stated are by no means the ultimate goal. However, they are a step in the right direction. There is much needed research to be conducted into various aspects of overland flow before this or any other method can be said to be the end product. Before any technique can be used in actual practice, a complete understanding of the rainfall runoff relationship must be understood and incorporated into the model. Such phenomena as variations of infiltration, depression storage and surface roughness from one subarea to another must be incorporated in the

watershed simulation model. Studies could and should be made to determine:

1. The effect of temporal and spatial variation of rainfall on outflow.
2. How the flow from one subarea combines with the flow of another area.
3. What constitutes the boundary for making a subarea.
4. How the flow from a given chain of subareas actually enters the main channel.

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APPENDIX II.--NOTATION

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The following symbols are used in this paper:

A	= Cross sectional area (BY)	ft <sup>2</sup>
A <sub>s</sub>	= Surface area (BAX)	ft <sup>2</sup>
B	= Width of element	ft
D	= Hydraulic mean depth	
F	= Hydrostatic pressure force	lb
F <sub>f</sub>	= Frictional force resisting flow	lb
F <sub>g</sub>	= Gravitational force in the direction of flow	lb
F <sub>N</sub>	= Froud number	
g	= Acceleration of gravity	ft/sec <sup>2</sup>
i	= Rainfall intensity	in/hr
K	= Conversion factor = (1.0/38,400.0)	ft hr/ in sec
L	= Length of subarea	ft
M	= Mass of fluid flowing (ρAΔX)	lb sec <sup>2</sup> /ft
N	= Manning's roughness coefficient	
P	= Wetted perimeter	ft
Q	= Flowrate	ft <sup>3</sup> /sec
R	= Hydraulic radius A/P	ft
S <sub>f</sub>	= The friction slope	ft/ft
S <sub>o</sub>	= Slope of watershed subarea	ft/ft
t	= Time	sec
V	= Average velocity of flow	ft/sec
W	= Weight of element = γAΔX	lb
ΔX	= Length of element (spacial increment)	ft

$Y$  = Depth of element ft

$\bar{Y}$  = Distance from surface to centroid of area ft

$\gamma$  = Specific weight lb/ft<sup>3</sup>

$\rho$  = Density of fluid lb sec<sup>2</sup>/ft<sup>4</sup>

## VITA

Glendon Taylor Stevens, Jr. was born on March 17, 1927, in Beaver Dam, Kentucky. He received his primary and secondary education in Beaver Dam, Kentucky. He received his college education from Kentucky Wesleyan, Owensboro, Kentucky; Western Kentucky University, Bowling Green, Kentucky; and the University of Missouri-Rolla, Rolla, Missouri. He received a Bachelor of Science degree in Civil Engineering from the University of Missouri-Rolla, Rolla, Missouri, in June 1964.

He has been enrolled in the Graduate School of the University of Missouri-Rolla since September 1964. He was the recipient of a teaching assistantship for the period September 1964 to June 1966. At that time he received a Master of Science degree in Civil Engineering. He has been an instructor in Civil Engineering from June 1966 until the present.

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